Black Hole Thermalization and Microstructure From Microstate Statistics

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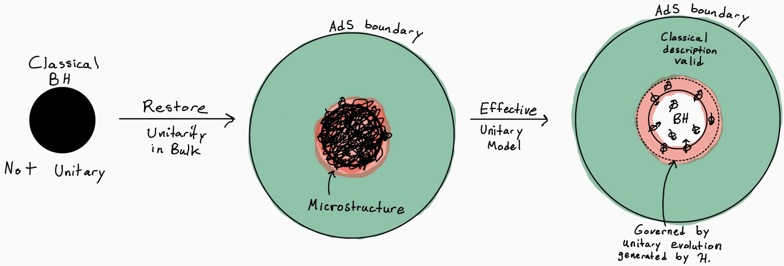
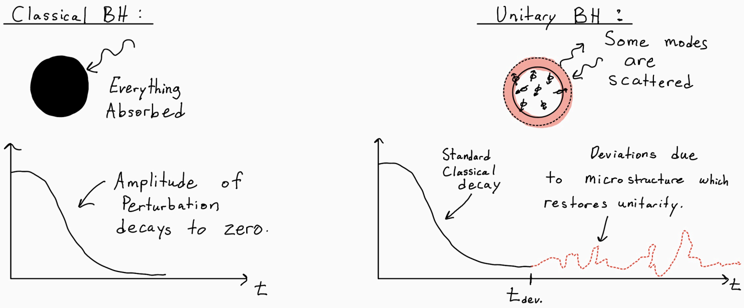
BH Unitarity and the need for Microstructure

- AdS/CFT formulates quantum gravity in AdS in terms of unitary CFT.
- BHs are thermal systems in CFT $\Rightarrow$ BH evolution is unitary.
- Unitarity is not manifest from classical bulk description.
- Motivates introduction of “microstructure” which restores unitarity.



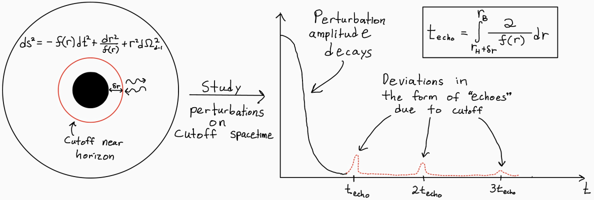
Ringdown of Classical vs Unitary BH

- There is a difference between classical and unitary BH thermalization.



- How and when do the deviations manifest?
- How is this related to details of the unitary description?

Echoes from Classical Models for BH Microstructure



- Cutoff has some semi-reflective boundary conditions.
- Echoes occur since perturbations repeatedly bouncing back and fourth between cutoff and outer boundary.
- $t_{echo} \sim t_{scrambling} \simeq \beta \ln(S)$ when cutoff is placed proper radial Planck length from horizon (KS & Afshordi 2019).
- **Should we generally expect to see deviations in the form of simple echoes?**

Unitary Thermalization and the Form Factor

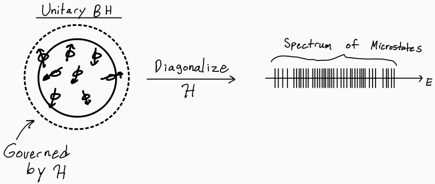
- View the black hole as a thermal ensmeble of $e^{S_{BH}}$ microstates.
- We are interested in the normalized form factor:

$$\frac{\mathcal{Z}(\beta + it)\mathcal{Z}(\beta - it)}{\mathcal{Z}(\beta)^2} = \frac{\sum_{n,m} e^{-\beta(E_m + E_n)} e^{i(E_n - E_m)t}}{\sum_{n,m} e^{-\beta(E_m + E_n)}} \tag{1}$$

- View form factor as proxy for 2-point function calculation in thermal ensemble of microstates.
- How is t_{dev} in form factor related to details of the spectrum of microstates?
- Are there echoes in the form factor?

BHs as Unitary Quantum Chaotic Systems

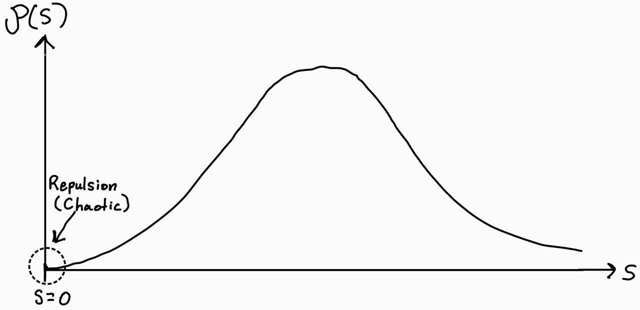
- Recent progresses in AdS/CFT suggest BHs are dual to quantum chaotic systems.
- Hamiltonian, $\mathcal{H}$, describes the dynamics of quantum chaotic system.



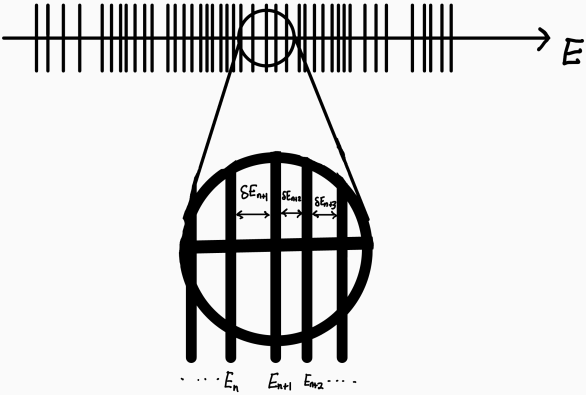
- Has consequences on spacing statistics between microstates.

Microstate Spacing Statistics and Quantum Chaos

- Generally quantum chaotic systems exhibit eigenvalue “repulsion.”
- $\mathcal{P}$ is probability density of spacing between two nearest neighbor pair of eigenvalues.



Statistical Model for Spectrum of BH Microstates

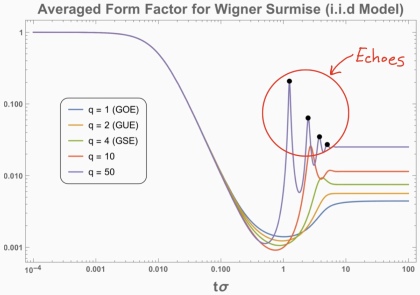


- Assume δE_k are random variables that are independent-identically-distributed (**i.i.d.** model) .
- Using i.i.d. model of random spectrum we have: $\mathcal{P} \mapsto \langle \mathcal{Z}(\beta + it)\mathcal{Z}(\beta - it)\rangle$.

Wigner Surmise Spacing Statistics

- Assume nearest neighbor spacings approximated by Wigner surmise:
$$\mathcal{P}_q(s) \sim s^q e^{-s^2} \tag{2}$$
- $\mathcal{P}_q(s=0) = 0 \Rightarrow$ Chaotic repulsion.
- $q = 1, 2,$ and 4 for classical Gaussian ensembles.
- We consider more general values of $q > 0$ which occur in β-ensembles.
- How does varying q affect thermalization behaviour?

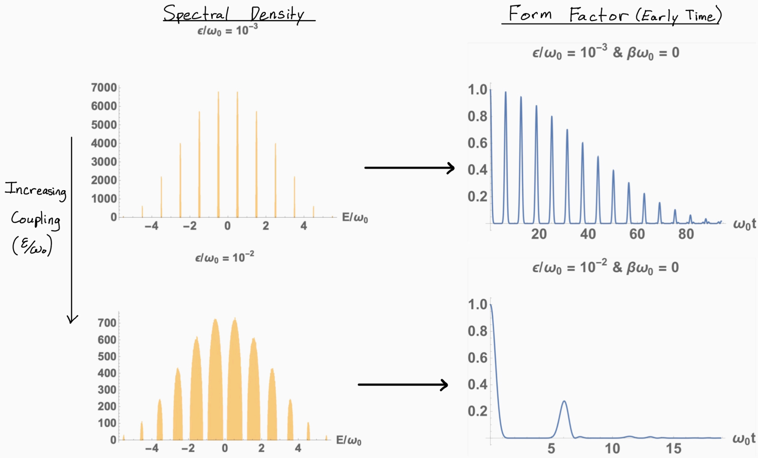
Wigner Surmise Form Factor



- As you increase q (i.e. repulsion) you start to see oscillations (echoes) before the plateau.
- Manifest on time scales $t \sim \langle \delta E \rangle^{-1} \sim t_{\mathrm{Heisenberg}} \gg t_{scrambling}$.
- Not quite the same as echoes in classical models.
- Is it possible to get more "classical" echoes? Yes, but you must violate i.i.d assumption.

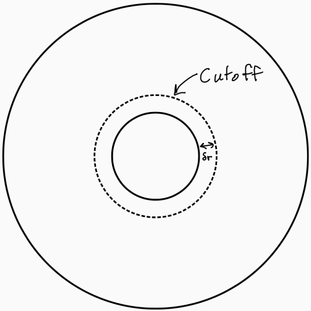
“Classical” Echoes from Separated Clusters of States

$$\mathcal{H} = \omega_0 \left[\bigoplus_{p=0}^{N/2} \left(p - \frac{N}{4} \right) \mathbb{I}_{\Omega(p)\times\Omega(p)} + \frac{\epsilon}{\omega_0} \mathcal{H}_{GUE} \right], \tag{3}$$



Coupled Oscillators as Toy Model of Unitary BH

Classical Cutoff Model



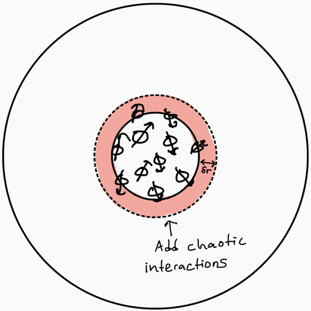
- Evenly Spaced Spectrum (normal modes)

$\Rightarrow$ Form factor oscillates

- No dissipation

$$\mathcal{H} = \omega_0 \mathcal{H}_{oscil}, \quad \omega_0 \sim \frac{2\pi}{t_{echo}}$$

"Quantum" Cutoff Model

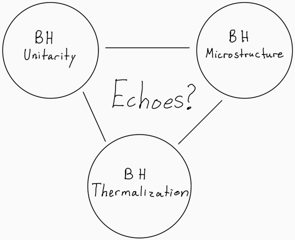


- Spectrum inherits chaotic statistics

$$\mathcal{H} = \omega_0 \left[\mathcal{H}_{oscil} + \frac{\varepsilon}{\omega_0} \mathcal{H}_{chaos} \right]$$

- ε/ω_0 depends on δr
- $\delta r \to 0 \quad \varepsilon/\omega_0 \to \infty$ (No echoes)
- $\delta r > 0 \quad \varepsilon/\omega_0 \to$ finite (Echoes)

Summary



- t_{dev} related to vicinity of microstructure to "horizon."
- For microstructure localized within proper Planck length of horizon, expect $t_{dev} \gtrsim \beta \ln(S) \sim t_{scrambling}$.
- Deviations in the form of **echoes** occur for systems with:
  - Enhanced eigenvalue repulsion (β-ensembles).
  - Regularly spaced cluster of states (coupled oscillator example).

Possibility of detecting imprints of microstructure in gravitational wave observations depends on the statistical properties of the black hole's spectrum of microstates.